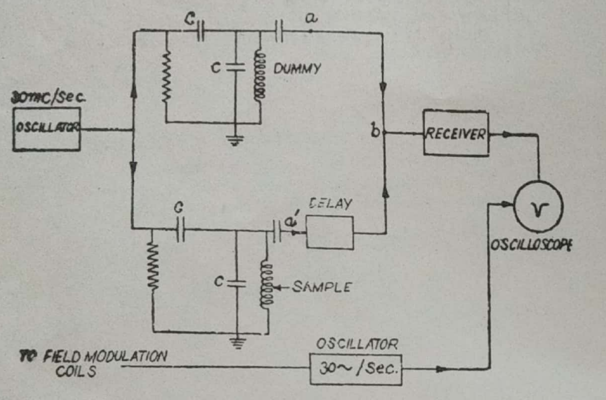


Fig. 3·23.

shown, whose output can be viewed on an oscilloscope can test whether or not there is exact cancellation of the two waves at b. If it is not so, this adjustment be obtained by varying the capacitors C.

Having achieved the cancellation, magnetic resonance condition is obtained by varying **H**. The resonance is obtained when there is net transfer of protons from their lower energy to higher energy position. At resonance, the wave form at a' is changed in both phase and amplitude by the added absorption in the sample. This destroys the belance at b and the resulting signal appears on the output oscilloscope. In practice **H** is modulated periodically at about 30 Hz by means of an alternating current sent through small, suitably placed field coils.

Once the proton moment is accurately known, the magnetic resonance technique can be used to measure magnetic field strengths. Because,

$$H = h\nu / 2\,\mu_P$$

where ν is the frequency corresponding to absorption and μ_P is the proton magnetic moment.

3·24. Electric Quadrupole Moment

The electric quadrupole moment measures the departure of a nucleus from spherical symmetrical. So far we have assumed that nuclei are spherically symmetrical. But it is not always necessary to make this supposition. Any nucleus in a stationary state does not possess a dipole moment because the centre of charge and centre of mass can be assumed to coincide with one another.

The electric quadroupole moment of a nucleus is calculated as follows from the classical considerations.

Let us consider that the charge is not situated at the centre of the nucleus (departure from spherical symmetry) but is located at P' having rectangular

co-orcinates (x, y, z). The potential at a point P on the Z - axis due to this charge is

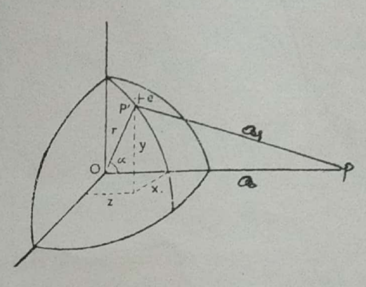


Fig. 3·24.

$$\phi_P = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e}{a_1} \qquad ...(3\cdot 65)$$

But

$$a_1 = (a^2 + r^2 - 2\,ar\cos\alpha)^{1/2} \qquad ...(3\cdot 66)$$

where r is the distance of the charge from the origin and is given by, $r = (x^2 + y^2 + z^2)^{1/2}$ and $\cos\alpha = \frac{z}{r}$ defines the angle between a and r.

Eq. (3· 65) now becomes

$$\phi_P = \frac{1}{4\pi\varepsilon_0}\frac{e}{(a^2 + r^2 - 2\,ar\cos\alpha)^{1/2}} = \frac{e}{a}\left(1 - 2\frac{r}{a}\cos\alpha + \frac{r^2}{a^2}\right)^{-1/2} \qquad ...(3\cdot 67)$$

$$\phi_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{e}{a} + \frac{e}{a^2}r\cos\alpha + \frac{e}{a^3}r^2\left(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2}\right) + \frac{er^3}{a^4}\left(\tfrac{5}{2}\cos^3\alpha - \tfrac{3}{2}\cos\alpha\right) + \ldots\ldots\right] \qquad ...(3\cdot 68)$$

or

$$\phi_P = \frac{1}{4\pi\varepsilon_0}\Sigma_{n=0}^{\infty}\frac{e\,r^n}{a^{n+1}}P_n(\cos\alpha) \qquad ...(3\cdot 69)$$

where $P_n(\cos\alpha)$ are the Legendre polynomials and n is the multipole order.

In the eq. (3· 68), the cofficient of $\frac{1}{a}$ is known as monopole strength, the coefficient of $\frac{1}{a^2}$ is the Z-component of the dipole moment, the coefficient of $\frac{1}{a^3}$ is the Z-component of quadrupole moment, the coefficient of $\frac{1}{a^4}$, is the Z-component of octupole moment etc. The first term in the above expression is the ordinary Coulomb potential.

Thus the nucleus possesses a net electric quadrupole moment given by

$$Q = e\cdot r^2\left(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2}\right)$$

$$= \frac{e r^2}{2} (3 \cos^2 \alpha - 1)$$

Putting $\cos \alpha = \frac{z}{r}$, we get

$$Q = \frac{e r^2}{2} \left(3 \frac{z^2}{r^2} - 1 \right)$$

$$Q = \frac{e}{2} (3z^2 - r^2) \qquad ...(3 \cdot 70)$$

The above expression shows that a nucleus will posses a finite quadrupole moment only when the protons are placed asymmetrically. However, if the arrangement of protons is symmetrical to three axes, the net quadrupole moment is zero. This is obvious from the following consideration.

Let us consider a single proton situated at the nuclear radius R along the body axis *i. e.* $z = r = R$ and $x = y = 0$; then the quadrupole moment from equation (3 ·70) is given by

$$Q = e R^2 \qquad ...(3 \cdot 71)$$

Similarly for a proton situated at nuclear equator $z = 0$, $r = R$, we have

$$Q = -\tfrac{1}{2} e R^2 \qquad ...(3 \cdot 72)$$

Thus, if six protons are located symmetrically along the co-ordinate axes at distance $\pm R$ from the centre, the two protons at $z = \pm R$ on the body axis of the angular momentum would contribute $+ 2 e R^2$ to the quadrupole moment while the four protons in the equatorial plane at $x = \pm R$ and $y = \pm R$ would contribute $4(-e R^2/2) = -2 e R^2$. Thus the total contribution from all the six protons would be $2 e R^2 - 2 e R^2 = 0$ and the nucleus will have zero quardrupole moment.

But if in addition to a symmetrical distribution, some protons are asymmetrically located, the nucleus will possess a net electric quadrupole moment. Nuclei can have possitive or negative quadrupole moments. Positive moments correspond to an alongation of the nuclear charge distribution along the angular momentum axis while negative moments correspond to flattened or oblate distribution.

Eq. (3 ·70) for quadrupole moment has been derived from classical considerations. When quantum mechanics is applied, quadrupole moment receives a new definition. The new definition differs from the classical one in the following respects.

(1) In classical consideration, the quardrupole moment is taken about the body axis of I whereas in quantum mechanics, it is not taken about the body axis but about the axis of its maximum projected componet $m_I = I$.

(2) The numerical factor $\frac{1}{2}$ in the classical expression disappears in the quantum mechanical expression.

* average value of $(\cos^2\alpha - 1)$ becomes zero. This does not mean that nuclei with $I = 1/2$ have spherical symmetry in charge distribution but only the maximum observable component of Q is zero.

mechnical charge distribution is, therefore, continous and can be represented by a mean charge density $\rho(x, y, z)$.

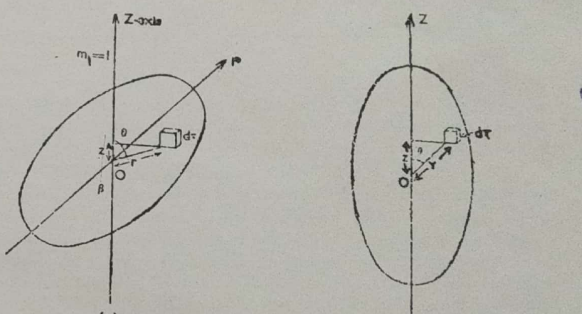
(4) The integral over the charge distribution is divided by the proton charge e which gives all nuclear quadrupole moments a dimension of cm^2 only.

Thus, if σ is the nuclear charge density in volume $d\tau$ at the point (z, r), the nuclear quadrupole moment will then be defined as

$$Q = \frac{1}{e} \int \sigma (3z^2 - r^2) \, d\tau$$

$$= \frac{1}{e} \int \sigma r^2 (3 \cos^2 \alpha - 1) \, d\tau \qquad ...(3 \cdot 73)$$

and is taken about $m_I = I$.



(a) Quantum-mechanical

(b) Classical limit

Fig. 3·25.

If β is the angle between the body axis I and the Z-axis in space, then $\cos \beta$ is given by

$$\cos \beta = \frac{m_I}{\sqrt{\{I(I+1)\}}} \qquad ...(3 \cdot 74)$$

where m_I is the magnetic quantum number.

It has been shown that the effective value of quadrupole moment Q is proportional to $(3 \cos^2 \beta - 1)$. Then the effective value $Q_{(m_I)}$ in the state m_I to the value of Q in the state $m_I = I$ is given as

$$\frac{Q_{(m_I)}}{Q} = \frac{3 \cos^2 \beta_{mI} - 1}{3 \cos^2 \beta_I - 1} = \frac{3 m_I^2 - I(I+1)}{I(2I-1)} \qquad ...(3 \cdot 75)$$

Nuclei which have $I = 0$ or $I = \frac{1}{2}$ will have no quadrupole moment in the state

$m_I = I$ For $I = \frac{1}{2}$, $\cos \beta_I = \frac{\frac{1}{2}}{\sqrt{\frac{1}{2} \times 3/2}} = \frac{1}{\sqrt{3}}$ and from symmetry consideration, an